Pearson Edexcel Level 3 GCE

Further Mathematics

Advanced Subsidiary
Further Mathematics options
27: Decision Mathematics 1
(Part of options D, F, H and K)

Thursday 17 May 2018 – Afternoon

Paper Reference
8FM0-27

You must have:
Mathematical Formulae and Statistical Tables, calculator, D1 Answer Book (enclosed)

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the Answer Book provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

P 6 0 2 0 8 A

Pearson

1.

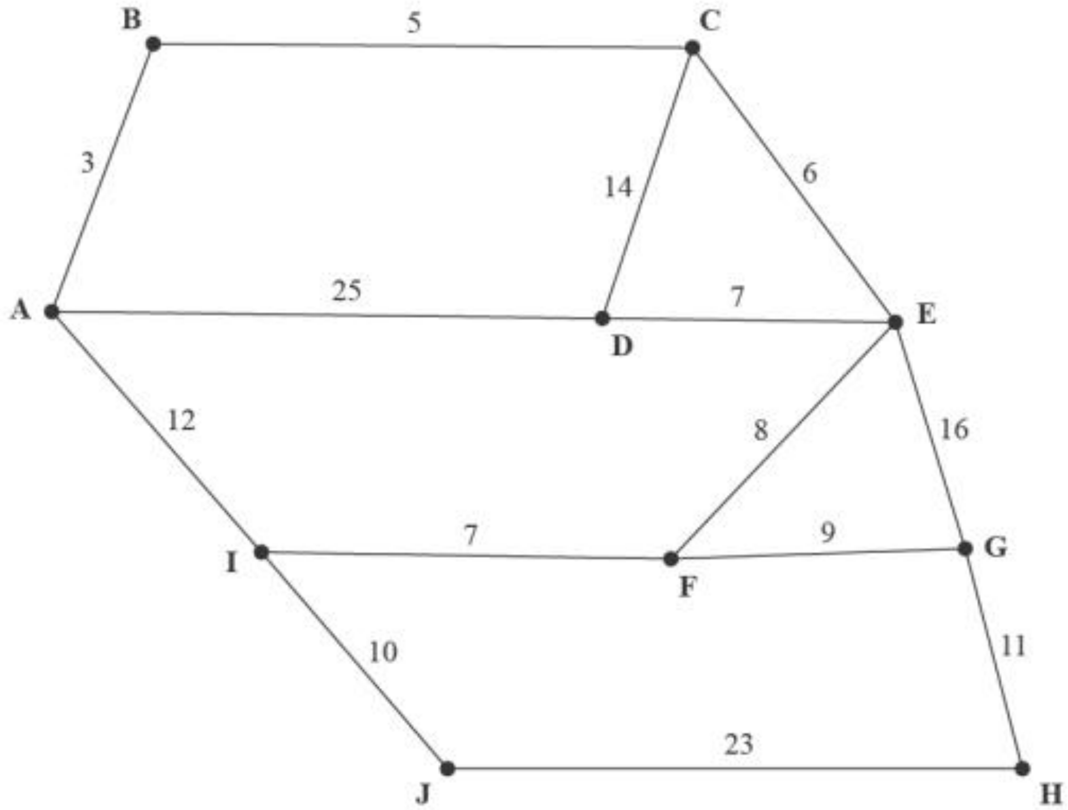


Figure 1

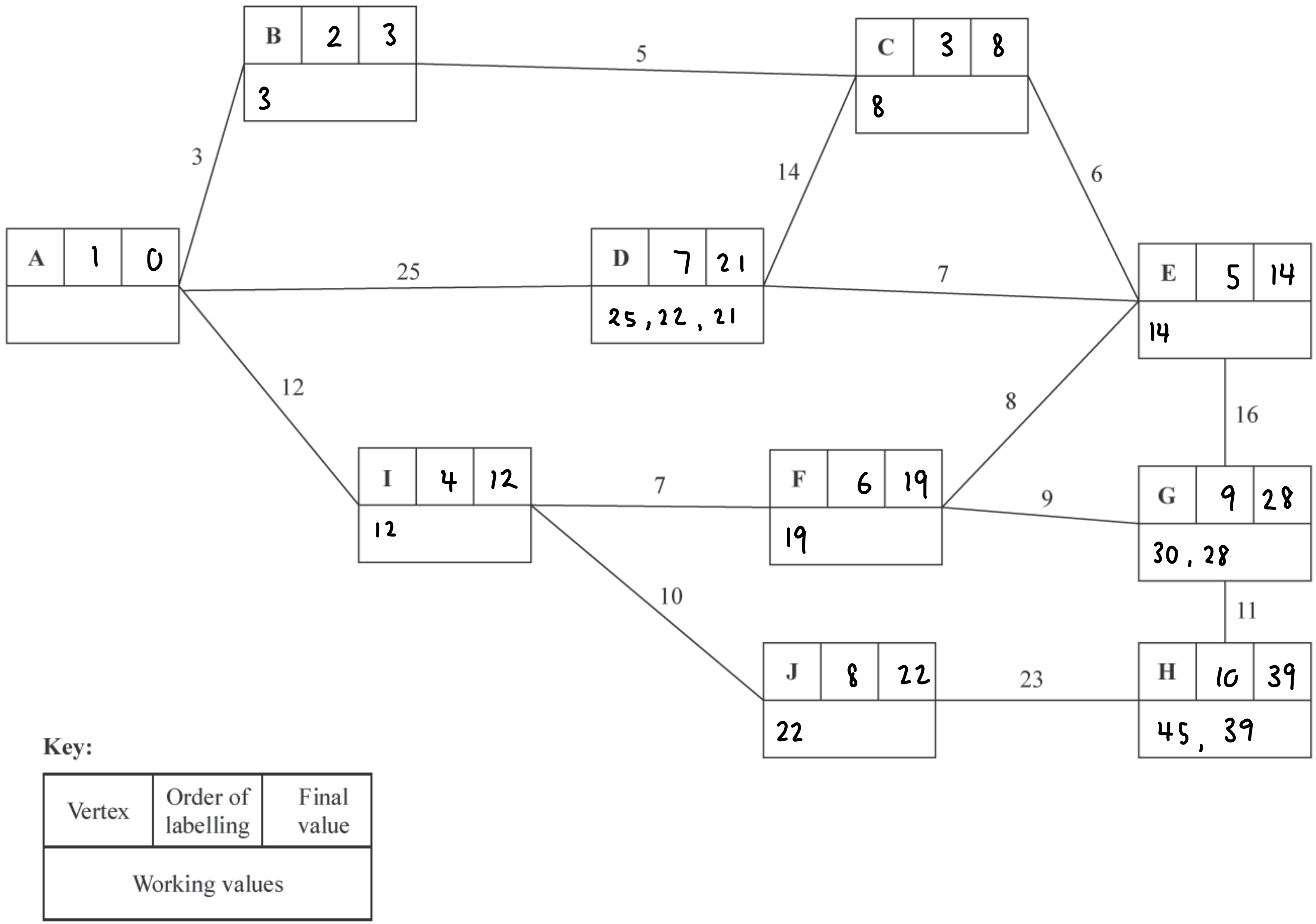
Figure 1 represents a network of roads.
The number on each arc represents the time taken, in minutes, to drive along the corresponding road.

(a) (i) Use Dijkstra's algorithm to find the shortest time needed to travel from A to H.

(ii) State the quickest route.

(6)

1.



Shortest time: 39 minutes

Quickest route: AIFGH

Leave blank

P 6 0 2 0 8 A 0 2 1 2

For a network with n vertices, Dijkstra's algorithm has order n^2

(b) If it takes 1.5 seconds to run the algorithm when $n = 250$, calculate approximately how long it will take, in seconds, to run the algorithm when $n = 9500$. You should make your method and working clear.

$250^2 \rightarrow 1.5\text{ s}$ $\quad \frac{9500^2}{250^2} \times 1.5 = 2166$ **(2)**

$9500^2 \rightarrow ?$

(c) Explain why your answer to part (b) is only an approximation.

Time taken to run algorithm might not be directly proportional to n^2 **(1)**

(Total for Question 1 is 9 marks)

---

2. A simply connected graph is a connected graph in which any two vertices are directly connected by at most one arc and no vertex is directly connected to itself.

(a) Given that a simply connected graph has exactly four vertices,

(i) write down the minimum number of arcs it can have,

(ii) write down the maximum number of arcs it can have.

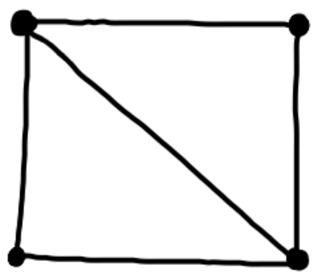
(2)

$n=4$

Minimum number of arcs: $4-1 = 3$

Maximum number of arcs: $\frac{4(4-1)}{2} = 6$

(b) (i) Draw a simply connected graph that has exactly four vertices and exactly five arcs.



(ii) State, with justification, whether your graph is Eulerian, semi-Eulerian or neither.

(3)

ii) There are 2 odd nodes so the graph is semi-Eulerian

(c) By considering the orders of the vertices, explain why there is only one simply connected graph with exactly four vertices and exactly five arcs.

(5)

(Total for Question 2 is 10 marks)

Question 2 continued

number of arcs

c) the sum of order of vertices : $5 \times 2 = 10$

possible orders at each vertex : ① 1 , 3 , 3 , 3

② 2 , 2 , 3 , 3

these are the only possible combinations as there are 4 vertices and in a connected graph the order at each vertex must be >0

① 3 of the vertices need to be connected to 3 vertices other than itself, but there are only 4 vertices and 1 is only connected by 1 arc

∴ this is not possible

② There is only one possible way to draw this graph as the vertices with an order of 3 need to be connected to all other vertices, leaving the remaining 2 vertices with an order of 2 each.

3.

Activity	Time taken (days)	Immediately preceding activities
A	5	–
B	8	–
C	4	–
D	14	A
E	10	A
F	3	B, C, E
G	7	C
H	5	D, F, G
I	7	H
J	9	H

The table above shows the activities required for the completion of a building project. For each activity, the table shows the time it takes, in days, and the immediately preceding activities. Each activity requires one worker. The project is to be completed in the shortest possible time.

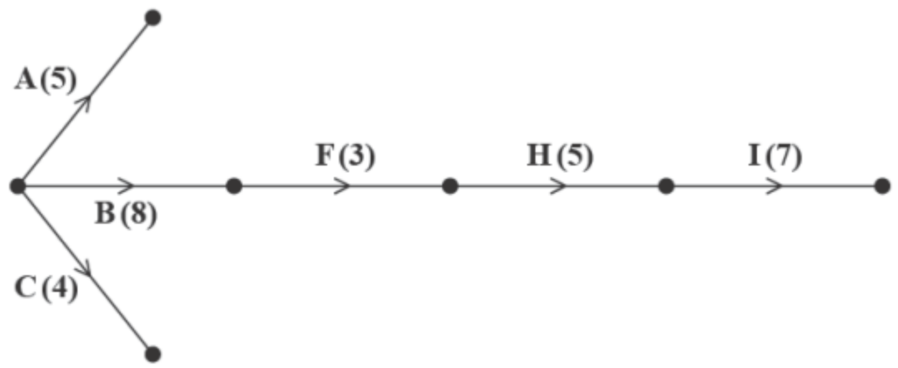


Figure 2

Figure 2 shows a partially completed activity network used to model the project. The activities are represented by the arcs and the number in brackets on each arc is the time taken, in days, to complete the corresponding activity.

(a) Add the missing activities and necessary dummies to Diagram 1 in the answer book.

(3)

(b) Complete Diagram 1 in the answer book to show the early event times and the late event times.

(3)

3.

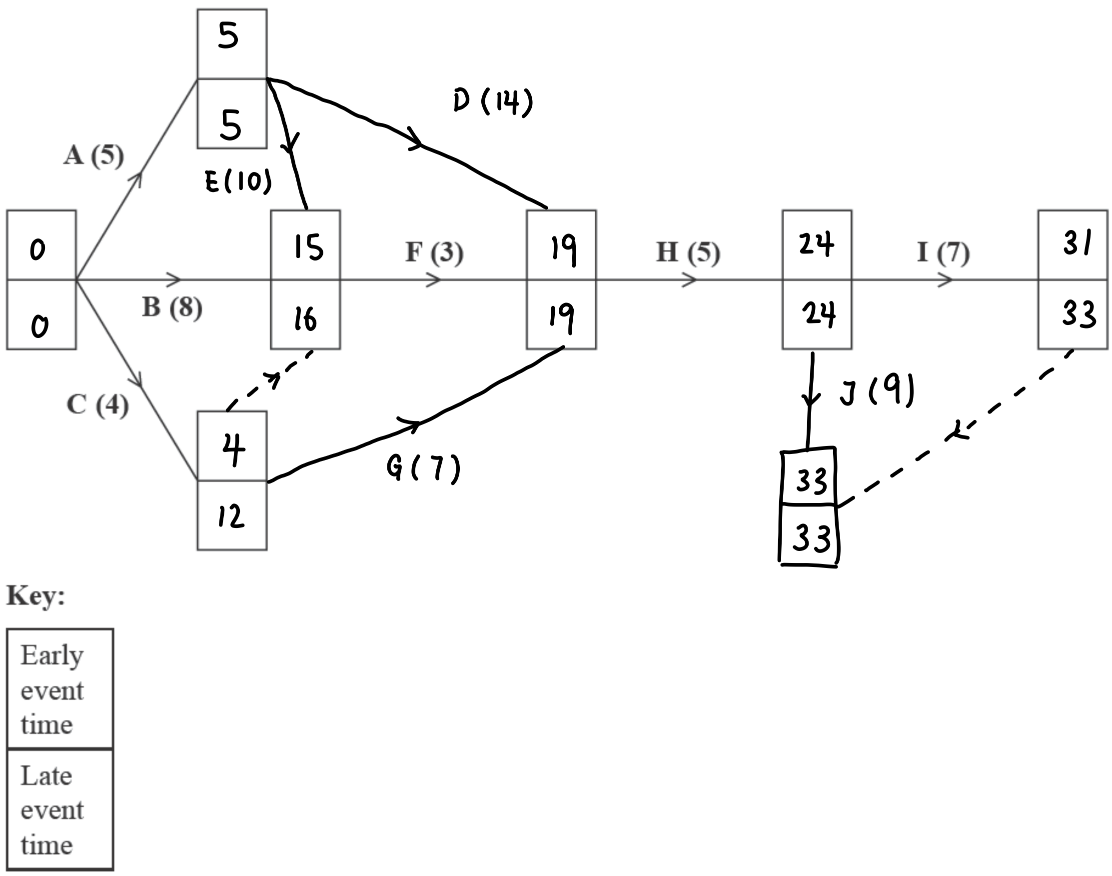


Diagram 1

c) A, D, H, J

P 6 0 2 0 8 A 0 8 1 2

Leave blank

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(c) State the critical activities.

A, D, H, J

(1)

At the beginning of the project it is decided that activity G is no longer required.

(d) Explain what effect, if any, this will have on

(i) the shortest completion time of the project if activity G is no longer required,

no effect as G is not a critical activity

(ii) the timing of the remaining activities.

activity C can start as late as day 12 and finish at day 16

(3)

(Total for Question 3 is 10 marks)

---

4. The manager of a factory is planning the production schedule for the next three weeks for a range of cabinets. The following constraints apply to the production schedule.

- The total number of cabinets produced in week 3 cannot be fewer than the total number produced in weeks 1 and 2
- At most twice as many cabinets must be produced in week 3 as in week 2
- The number of cabinets produced in weeks 2 and 3 must, in total, be at most 125

The production cost for each cabinet produced in weeks 1, 2 and 3 is £250, £275 and £200 respectively.

The factory manager decides to formulate a linear programming problem to find a production schedule that minimises the total cost of production.

The objective is to minimise $250x + 275y + 200z$

(a) Explain what the variables x, y and z represent.

no. of cabinets produced in week 1 (x), week 2 (y) and week 3 (z)

(1)

(b) Write down the constraints of the linear programming problem in terms of x, y and z.

(2)

b) $x, y, z \geq 0$

$z \geq x + y$

$z \leq 2y$

$y + z \leq 125$

Due to demand, exactly 150 cabinets must be produced during these three weeks. This reduces the constraints to

$$x + y \leqslant 75$$
$$x + 3y \geqslant 150$$
$$x \geqslant 25$$
$$y \geqslant 0$$

which are shown in Diagram 1 in the answer book.

Given that the manager does not want any cabinets left unfinished at the end of a week,

(c) (i) use a graphical approach to solve the linear programming problem and hence determine the production schedule which minimises the cost of production. You should make your method and working clear.

ci) $x+y+z=150 \Rightarrow z=150-x-y$

objective : minimise $C = 250x + 275y + 200(150-x-y)$

$= 30000 + 50x + 75y$

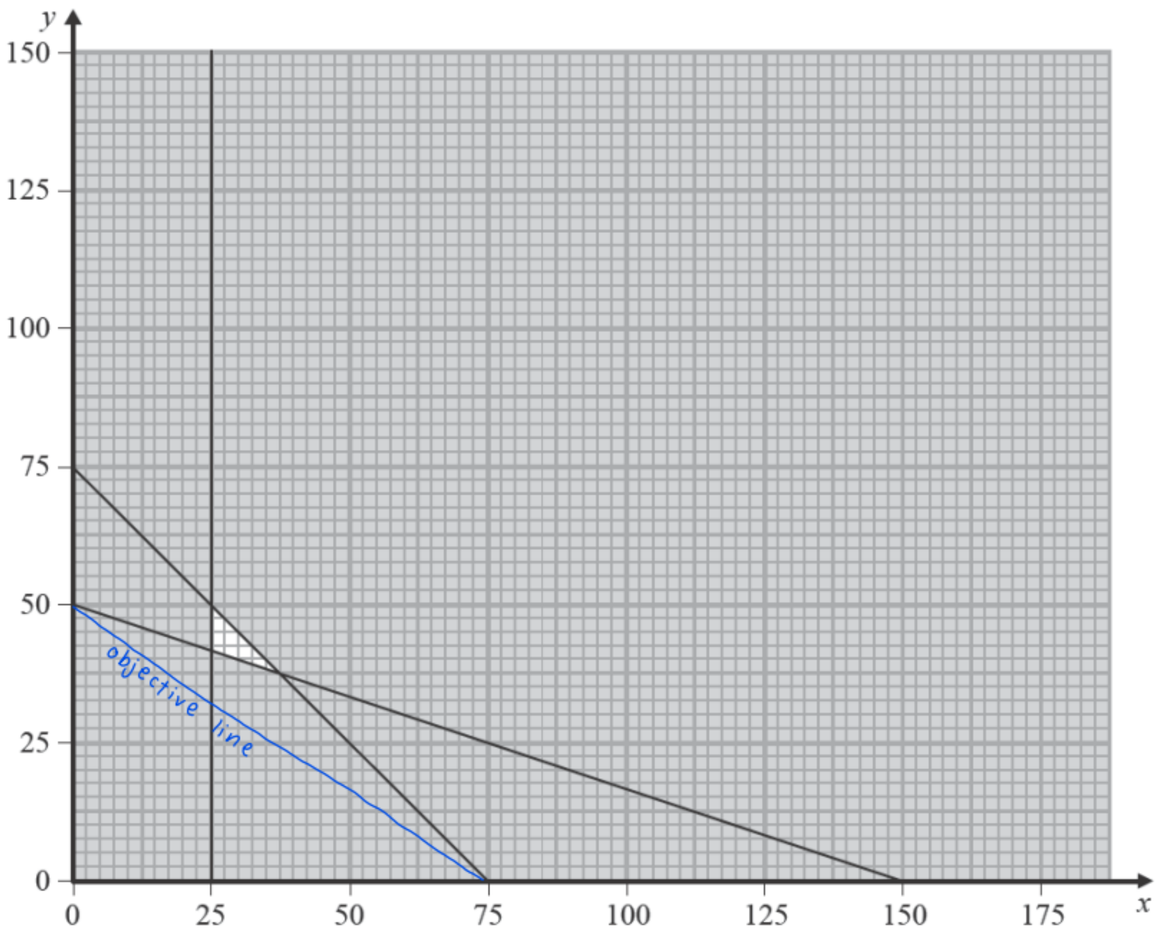


Diagram 1

(ii) Find the minimum total cost of the production schedule.

$250(25) + 275(42) + 200(83) = £34400$

(8)

(Total for Question 4 is 11 marks)